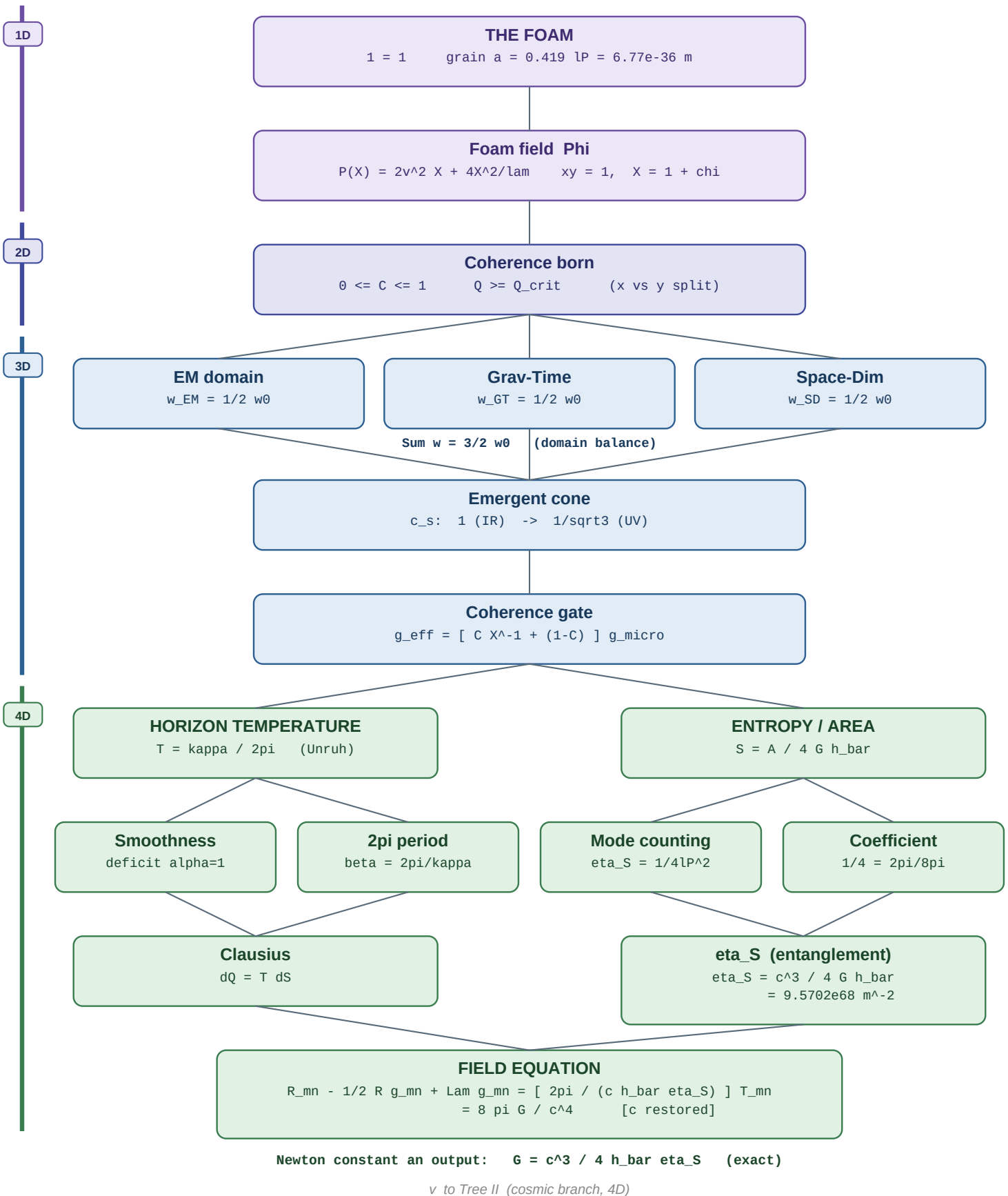


# THE FOAM TREE - I

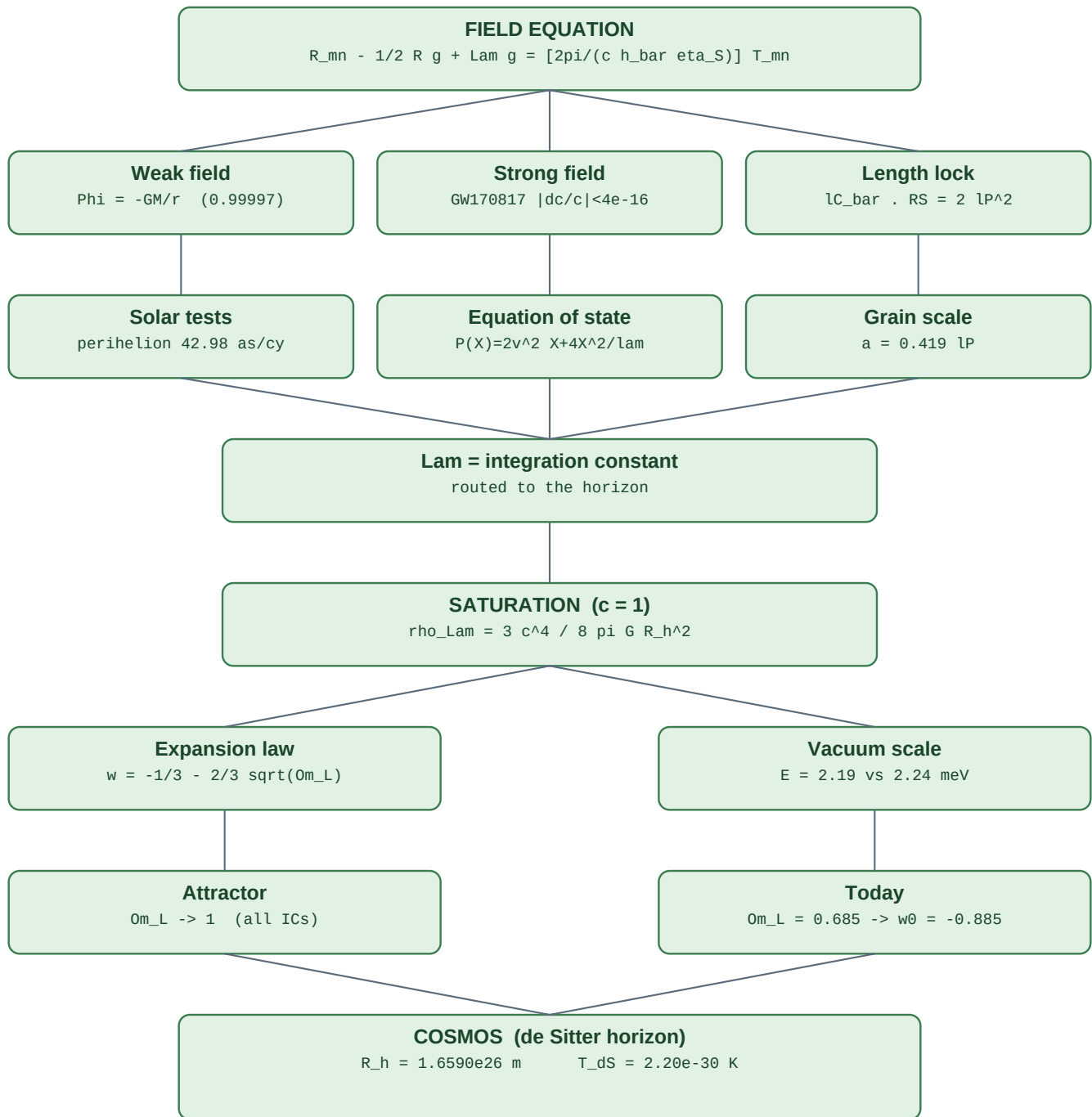
foam root -> field equation (coloured by dimension)



# THE FOAM TREE - II

field equation -> cosmos (all 4D)

4D



falsifiable (scalar sector):  $1 - v_s/c = A (E/E_g)^2$  photons ride the metric cone

foam root ---> cosmic canopy : one field, one grain

# THE FOAM TREE - III

4D field equation -> the lift -> the cap

4D

5D

6D

7D

**Field equation (Trees I / II)**  
 $G_{mn} + \Lambda g_{mn} = [2\pi/(c \hbar \eta_S)] T_{mn}$

**THE LIFT**  
 $ds^2 = g_{mn} dx^m dx^n + \phi^2 (dy + l A_m dx^m)^2$   
mode split at  $D=5$  :  $5 = 2 + 2 + 1$

**Kaluza sector**  
 $A_m = g_{m5} / l \phi^2$

**Dilaton**  
 $\phi = X$  ,  $G_{eff} = G(X)$

**Coherence axis**  
 $C = C(y)$

**Radius lock**  
 $R = 2 l_P / \sqrt{\alpha}$

**Modulus frozen**  
 $m > 5.112 \text{ meV}$

**Warp theorem**  
 $2k|y| = C \chi$

**KK tower**  
 $5.2147e17 \text{ GeV}$

**Wesson / STM**  
hosts  $\Delta$ , selects none

**Empirical fence**  
 $D = 4.02$  ,  $ISL \ 38.6 \text{ um}$

**Gauss-Bonnet : grain term suppressed**  
 $(a/r)^2 \leq 1.70e-62 \text{ at } 52 \text{ um}$

**REDUCTION LOCK 5F**  
slice  $y_0$  returns  $g_{eff}$  , field equation , clock

**Counting ladder**  
 $N(D) = D(D-3)/2 = 2 + 2n + n(n+1)/2$  ,  $D = 4 + n$

**6D rung**  
 $9 = 2 + 4 + 3$   
self-dual 2-form ; 3 moduli, 3 Cassini liabilities

**7D rung**  
 $14 = 2 + 6 + 6$   
6 vectors, 6 moduli

**Isometry floor**  
 $\dim(G/H) = 7 \rightarrow D = 11$   
(terminus, above the 7-rung palette)

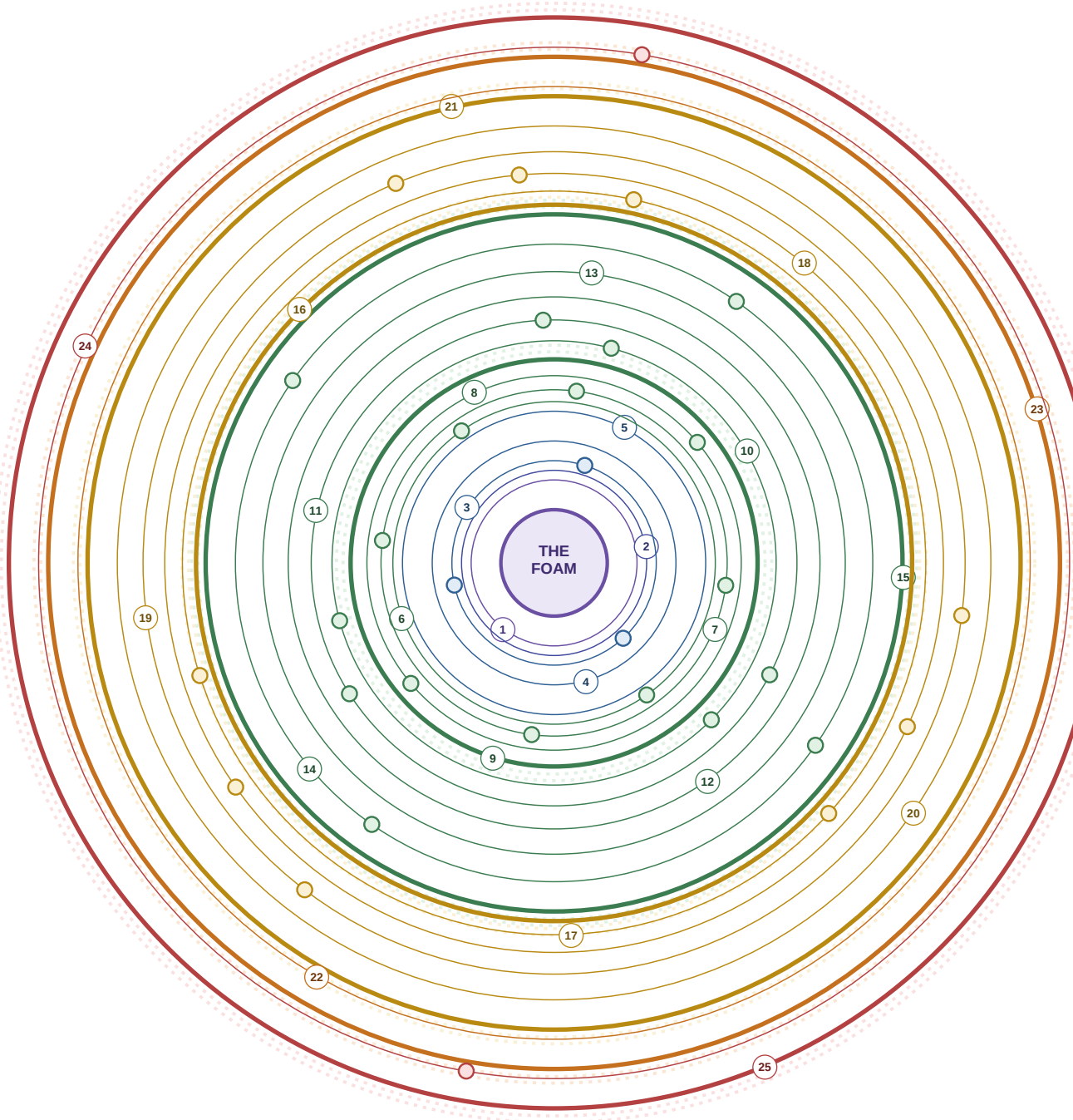
**CAP : chirality obstruction**  
smooth manifold + continuous isometry -> no chiral fermions

the ladder terminates : the tree's own smoothness node blocks the orbifold repair

all coefficients c-restored; conventions and symbol table in Appendix A  
ADD, warped-hierarchy and self-accelerating-brane branches are excluded and are not carried

# THE FOAM TREE - IV

one field, one grain : the whole tree as a pulsation - rings widen through a dimension, then burst into the next



| SHELL                              | RELATION                                                   |
|------------------------------------|------------------------------------------------------------|
| 00 THE FOAM (nucleus)              | $1 = 1$ , $a = 0.419 \text{ LP}$                           |
| 01 Foam field Phi                  | $P(X) = 2v^2 X + 4X^2/\lambda m$                           |
| 02 Coherence born                  | $0 \leq C \leq 1$ , $Q \geq Q_{\text{crit}}$               |
| 03 EM / Grav-Time / Space-Dim      | $w = 1/2 w_0 \text{ each}$ , $\text{sum} = 3/2 w_0$        |
| 04 Emergent cone                   | $c_s : 1 \text{ (IR)} \rightarrow 1/\sqrt{3} \text{ (UV)}$ |
| 05 Coherence gate                  | $g_{\text{eff}} = [C X^{-1} + (1-C)] g_{\text{micro}}$     |
| 06 Horizon T / Entropy-Area        | $T = \kappa/2\pi$ ; $S = A/4G \hbar$                       |
| 07 Smoothness / $2\pi$ / Modes     | $\eta_S = 1/4LP^2$ ; $1/4 = 2\pi/8\pi$                     |
| 08 Clausius / $\eta_S$             | $dQ = T dS$ ; $\eta_S = c^3/4G \hbar$                      |
| 09 FIELD EQUATION *burst*          | $G + \lambda m g = [2\pi/(c \hbar \eta_S)] T$              |
| 10 Weak / Strong / Length lock     | $0.99997$ ; $4e-16$ ; $1C.RS = 21P^2$                      |
| 11 Solar / Eq of state / Grain     | $42.98 \text{ as/cy}$ ; $a = 0.419 \text{ LP}$             |
| 12 Lam = integration constant      | routed to the horizon                                      |
| 13 SATURATION                      | $\rho_{\text{Lam}} = 3c^4 / 8 \pi G R_h^2$                 |
| 14 Expansion / Vacuum / Attractor  | $w = -1/3 - 2/3 \sqrt{0m_L} = -0.885$                      |
| 15 COSMOS *burst*                  | $R_h = 1.659e26 \text{ m}$ , $T = 2.20e-30 \text{ K}$      |
| 16 THE LIFT *burst*                | $ds^2 = g + \phi^2 (dy + l A)^2$                           |
| 17 Kaluza / Dilaton / C-axis       | $A_m = g_{m5}/l \phi^2$ ; $\phi = X$                       |
| 18 Radius lock / Modulus / Warp    | $R = 21P/\sqrt{\alpha}$ ; $2k y  = C \chi$                 |
| 19 KK tower / Wesson / Fence       | $5.21e17 \text{ GeV}$ ; $D = 4.02$ , $38.6 \text{ um}$     |
| 20 Gauss-Bonnet : grain suppressed | $(a/r)^2 \leq 1.70e-62 \text{ at } 52 \text{ um}$          |
| 21 REDUCTION LOCK 5F *burst*       | slice $y_0$ returns the 4D tree                            |
| 22 Counting ladder                 | $N = 2 + 2n + n(n+1)/2$                                    |
| 23 6D rung *burst*                 | $9 = 2 + 4 + 3$                                            |
| 24 7D rung / Isometry floor        | $14 = 2+6+6$ ; $\dim(G/H)=7 \rightarrow D=11$              |
| 25 CAP : chirality obstruction     | smooth isometry $\rightarrow$ no chirality                 |

solid ring = single relation | dots = parallel relations on one shell | heavy + halo = burst into the next dimension  
shell spacing widens with position inside each dimension and resets after every burst : the wave, drawn

# THE FOAM TREE - COMPLETE

foam root -> field equation -> cosmos -> the lift -> the cap    one continuous flow

1D

**THE FOAM**  
 $1 = 1$     grain  $a = 0.419 \text{ LP} = 6.77\text{e-}36 \text{ m}$

2D

**Foam field Phi**  
 $P(X) = 2v^2 X + 4X^2/\text{lam}$      $xy = 1, \quad X = 1 + \text{chi}$

3D

**Coherence born**  
 $0 \leq C \leq 1$      $Q \geq Q_{\text{crit}}$     (x vs y split)

**EM domain**     $w_{\text{EM}} = 1/2 w\theta$     **Grav-Time**     $w_{\text{GT}} = 1/2 w\theta$     **Space-Dim**     $w_{\text{SD}} = 1/2 w\theta$

**Emergent cone**  
 $c_s : 1 \text{ (IR)} \rightarrow 1/\sqrt{3} \text{ (UV)}$

**Coherence gate**  
 $g_{\text{eff}} = [C X^4 + (1-C)] g_{\text{micro}}$

4D

**HORIZON TEMPERATURE**     $T = \kappa / 2\pi$     (Unruh)    **ENTROPY / AREA**     $S = A / 4 G \hbar$

**Smoothness**    deficit  $\alpha = 1$     **2pi period**     $\beta = 2\pi / \kappa$     **Mode counting**     $\eta_S = 1 / 4 \text{ LP}^2$     **Coefficient**     $1/4 = 2\pi / 8\pi$

**Clausius**     $dQ = T dS$     **eta\_S (entanglement)**     $c^3/4G \hbar = 9.57\text{e}268 \text{ m}^{\wedge}2$

**FIELD EQUATION**  
 $R_{mn} - 1/2 R g + \text{Lam} g = [2\pi / (c \hbar \eta_S)] T_{mn}$

**Weak field**     $\Phi = -GM/r$      $0.99997$     **Strong field**     $GW170817$      $|dc/c| < 4\text{e-}16$     **Length lock**     $LC_{\text{bar}} \cdot RS = 2 \text{ LP}^2$

**Solar tests**    perihelion  $42.98 \text{ as/cy}$     **Equation of state**     $P(X) = 2v^2 X + 4X^2/\text{lam}$     **Grain scale**     $a = 0.419 \text{ LP}$

**Lam = integration constant**  
routed to the horizon

**SATURATION (c = 1)**  
 $\rho_{\text{Lam}} = 3 c^4 / 8 \pi G R_{\text{h}}^2$

**Expansion law**     $w = -1/3 - 2/3 \sqrt{0m_L}$     **Vacuum scale**     $E = 2.19 \text{ vs } 2.24 \text{ meV}$

**Attractor**     $0m_L \rightarrow 1$     (all ICs)    **Today**     $0m_L = 0.685 \rightarrow w\theta = -0.885$

**COSMOS (de Sitter horizon)**  
 $R_h = 1.659\text{e}26 \text{ m}$      $T_{dS} = 2.2\text{e-}30 \text{ K}$

5D

**THE LIFT**  
 $ds^2 = g_{mn} dx^m dx^n + \phi^2 (dy + l A_m dx^m)^2$

**Kaluza sector**     $A_m = g_{m5} / l \phi^2$     **Dilaton**     $\phi = X, \quad G_{\text{eff}} = G(X)$     **Coherence axis**     $C = C(y)$

**Radius lock**     $R = 2 \text{ LP} / \sqrt{\alpha}$     **Modulus frozen**     $m > 5.112 \text{ meV}$     **Warp theorem**     $2k|y| = C \chi$

**KK tower**     $5.2147\text{e}17 \text{ GeV}$     **Wesson / STM**    hosts Delta, selects none    **Empirical fence**     $D = 4.02, \quad ISL 38.6 \text{ um}$

**Gauss-Bonnet : grain term suppressed**  
 $(a/r)^2 \leq 1.7\text{e-}62$     at  $52 \text{ um}$

**REDUCTION LOCK 5F**  
slice  $y\theta$  returns  $g_{\text{eff}}$ , field equation, clock

6D

**Counting ladder**  
 $N(D) = D(D-3)/2 = 2 + 2n + n(n+1)/2, \quad D = 4 + n$

**6D rung**  
 $9 = 2 + 4 + 3$     self-dual 2-form, 3 moduli

7D

**7D rung**     $14 = 2 + 6 + 6$     **Isometry floor**     $\text{dim}(G/H) = 7 \rightarrow D = 11$

**CAP : chirality obstruction**  
smooth manifold + continuous isometry -> no chiral fermions

falsifiable (scalar sector):  $1 - v_s/c = A (E/E_g)^2$     terminus: the tree's own smoothness node blocks the orbifold repair

DIMENSION    1D    2D    3D    4D    5D    6D    7D

colour advances only forward through dimension; every relation shown is stated in the appendix with its derivation

# APPENDIX A - conventions, symbols, and the action

read this page first; every relation in Trees I to IV is written in these conventions

## A0 UNITS

All relations are written in  $\hbar = c = k_B = 1$  unless a factor is shown explicitly.  
 SI is restored only in observational lines, and is then shown in full.  
 Energy density and pressure carry  $[P] = [\rho] = E^4$ . The frequency-native rendering  
 $Q = \text{number} \times \text{Hz}^n$  is a display convention for the same quantities, not a dimension.

## A1 SYMBOLS

|        |                         |                                                                                       |
|--------|-------------------------|---------------------------------------------------------------------------------------|
| a      | grain length            | $a = 0.419 \text{ lP} = 6.7721\text{e-}36 \text{ m}$                                  |
| kappa  | surface gravity         | $T = \kappa/2\pi$ SI: $T = \hbar \kappa / 2 \pi c k_B$                                |
| X      | foam field (modulus)    | background value written $X = 1 + \chi$ , $ \chi  \ll 1$                              |
| X_kin  | kinetic invariant       | $X_{\text{kin}} = -(1/2) g^{\text{mn}} d_m X d_n X$ , $x = X_{\text{kin}} / \mu^4$    |
| C      | coherence               | $0 \leq C \leq 1$                                                                     |
| F(X,C) | coherence gate          | $F = C/X + (1-C)$ ; matter metric $g_{\text{eff}} = F g$                              |
| Om     | conformal factor        | $Om^2 = F$ , $w = \ln Om$                                                             |
| eta_S  | horizon entropy density | $\eta_S = c^3/4G \hbar = 1/4 \text{ lP}^2 = 9.5702\text{e}68 \text{ m}^{-2}$          |
| mu^4   | substrate density       | $\mu^4 = \hbar c / a^4 = 1.5031\text{e}115 \text{ J/m}^3 = 32.44 \text{ rho\_Planck}$ |
| R      | fibre radius            | $R = 2 \text{ lP} / \sqrt{\alpha} = 23.4125 \text{ lP} = 3.7841\text{e-}34 \text{ m}$ |

## A2 the action

4D

$$S = \int d^4x \sqrt{-g} \left[ \left( M_P^2/2 \right) R + \mu^4 P(x) - (k_C/2)(dC)^2 - V(X,C) \right] + S_m[\psi, F g]$$

$P(x) = 2 v^2 x + 4 x^2 / \text{lam}$  is the kinetic sector of X, written non-canonically  
 $F(X,C) = C/X + (1-C)$  the gate, carried in the MATTER coupling only

Einstein-Hilbert is kept pure, so the curvature sector is ghost-free by construction and the frame is the micro frame.  $C = 0$  returns General Relativity exactly;  $C = 1$  gives a conformally coupled scalar with  $F = 1/X$ ; X and C constant give GR with a rescaled matter coupling and no new interaction.

## A3 the field equations

4D

$$\begin{aligned} \text{metric} \quad G_{mn} &= (1/M_P^2) \left[ T^\wedge(X)_{mn} + T^\wedge(C)_{mn} + F T^\wedge(m)_{mn} \right] \\ X \quad d/dX \left[ \mu^4 P \right] &= V_X - (1/2) F F_X T^\sim \\ C \quad k_C \left[ \right] C &= V_C - (1/2) F F_C T^\sim \end{aligned}$$

the matter power follows from  $\sqrt{-g_{\text{hat}}} = F^2 \sqrt{-g}$ ,  $g_{\text{hat}}^{\text{ab}} = F^{-1} g^{\text{ab}}$ , giving  
 $T^{\text{mn}} = F^3 T^{\sim\text{mn}}$  and  $T_{mn} = F T_{\sim mn}$ .  $T^\sim$  is the trace taken with  $g_{\text{eff}}$ .

## APPENDIX B - the 4D spine

the gate, the correction tensor, the two cones, the scalar sound speed, and the weak field

### B1 the coherence gate is the reciprocal identity

2D

root axiom  $X \cdot Y = 1 \Rightarrow Y = X^{-1}$   
gate  $g_{\text{eff}} = [C X^{-1} + (1-C)] g_{\text{micro}} = [C Y + (1-C) \cdot 1] g_{\text{micro}}$   
the gate is a C-weighted interpolation between the dual Y and unity: C = 1 gives the pure dual, C = 0 gives unity. C is the mixing angle between the reciprocal pair and identity.

### B2 the correction tensor, in closed form

3D

with  $0m^2 = F$ ,  $w = \ln 0m$ ,  $g_{\text{eff}} = 0m^2 g$ , the 4D conformal identity (Wald D.9) gives  
 $R_{mn} = R_{mn} - 2(D_m D_n w - D_m w D_n w) - g_{mn} (w + 2(Dw)^2)$   
 $R_{mn} = 0m^2 [R_{mn} - 6(Dw)^2]$   
contracting,  $G_{mn} = G_{mn} + \Delta_{mn}$ , with

$$\Delta_{mn}[X, C] = -2 D_m D_n w + 2 D_m w D_n w + 2 g_{mn} (w + g_{mn} (Dw)^2)$$

w is a scalar, so the identity holds for arbitrary w and arbitrary g; the X and C dependence enters on expanding  $D_m w = w_X D_m X + w_C D_m C$  and is already contained.  
symbolic check on  $g = e^{2f} \eta$  with  $f = f(x)$  returns  $G_{mn} - \Delta_{mn} = 0$  exactly:  
 $\Delta_{00} = -2f'' - f'^2$ ,  $\Delta_{11} = 3 f'^2$ ,  $\Delta_{22} = 2f'' + f'^2$

### B3 photons and tensor modes share one cone

3D

a conformal rescaling leaves the null condition invariant:  
 $g_{mn} k^m k^n = 0 \Leftrightarrow 0m^2 g_{mn} k^m k^n = 0$  for 0m finite and non-zero  
so null geodesics of  $g_{\text{eff}}$  and  $g_{\text{micro}}$  coincide for any C(x), X(x): the photon and the tensor mode ride one cone identically. GW170817 ( $|dc/c| < 4e-16$ ) is met structurally.  
the scalar sound speed below is a property of a different mode and does not enter here.

### B4 scalar sound speed and stability

3D

$P(x) = 2 v^2 x + 4 x^2 / \lambda m$   $P_x = 2v^2 + 8x/\lambda m$   $P_{xx} = 8/\lambda m$   
 $c_s^2 = P_x / (P_x + 2 x P_{xx}) = (\lambda m v^2 + 4x) / (\lambda m v^2 + 12x)$   
 $x \rightarrow 0 : c_s^2 \rightarrow 1$   $x \rightarrow \infty : c_s^2 \rightarrow 1/3$   $c_s^2 = 1/2$  at  $x = \lambda m v^2/4$   
no ghost  $P_x = 2v^2 + 8x/\lambda m > 0$  identically for x,  $\lambda m, v^2 > 0$   
no gradient  $P_x + 2x P_{xx} = 2v^2 + 24x/\lambda m > 0$  identically  
SCOPE: the  $1/\sqrt{3}$  asymptote is reached at x above the substrate scale  $\mu^4$ , where the effective description is the lattice rather than P(x). It is a property of the polynomial and of the lattice measurement (0.600 against 0.585 in the dense phase), and is quoted as such. The stability conditions hold on the whole physical sheet regardless.

### B5 the weak field, and the sign of the foam force

3D

matter follows geodesics of  $g_{\text{eff}}$ , so  $g_{\text{eff},00} = -0m^2 (1 + 2\Phi/c^2)$  gives  
 $\Phi_{\text{eff}} = \Phi_{\text{micro}} + c^2 \ln 0m$  and for C = 1,  $\Phi_{\text{foam}} = -(c^2/2) \ln X$   
the sign follows from the matter coupling. For dust,  $T = -\rho$ , and  $F F_X = -1/X^3$ , so  
source term =  $-\rho / (2 X^3) < 0 \Rightarrow \text{Laplacian}(X) < 0$  at the mass  
X therefore has a local maximum on matter: X is the compression variable, and

$$g_{\text{foam}} = + (c^2/2) C \text{ grad } \ln X \quad \text{attraction}$$

the effective potential deepens at the source, as required.

## APPENDIX C - cosmology and the solar system

the expansion law, its attractor, the post-Newtonian limit, and the horizon numbers

### C1 the expansion law from horizon saturation

4D

saturation :  $\rho_{\text{Lam}} = 3 M_{\text{P}}^2 / L^2$  ,  $L = R_{\text{h}} = a \int_{t_0}^{\infty} dt'/a$  ( future event horizon )  
definition :  $0_{\text{m}}L = \rho_{\text{Lam}} / (3 M_{\text{P}}^2 H^2) = 1/(LH)^2 \Rightarrow L H = 0_{\text{m}}L^{-1/2}$   
differentiate  $\rho_{\text{Lam}}' = -6 M_{\text{P}}^2 \dot{L} / L^3$   
conservation  $\rho_{\text{Lam}}' = -3H(1+w) \rho_{\text{Lam}} = -9 M_{\text{P}}^2 H (1+w) / L^2$   
equate  $2 \dot{L} / L = 3 H (1+w) \Rightarrow 1+w = (2/3) \dot{L}/(HL)$   
kinematics  $\dot{L} = H L - 1 \Rightarrow \dot{L}/(HL) = 1 - \sqrt{0_{\text{m}}L}$

$$w = -1/3 - (2/3) \sqrt{0_{\text{m}}L} \quad w(0.685) = -0.885098$$

the leading  $-1/3$  is the causal-cone term  $1/d$  at  $d = 3$ . Saturation is a postulate of the framework and is carried as one: the law is what saturation implies, exactly and with no free parameter, which is what makes it falsifiable against  $w(z)$  from supernovae and BAO.

### C2 the late-time attractor

4D

flat, matter plus vacuum,  $N = \ln a$  :  $d0_{\text{m}}/dN = 3 0_{\text{m}} (1-0_{\text{m}})(w_{\text{m}} - w)$  ,  $w_{\text{m}} = 0$   
 $F(0_{\text{m}}) = 0_{\text{m}} (1-0_{\text{m}})(1 + 2 \sqrt{0_{\text{m}}})$   
fixed points  $0_{\text{m}} = 0$  and  $0_{\text{m}} = 1$  , with no interior root on  $(0,1)$   
linearise  $F'(0) = +1$  ( repeller )  $F'(1) = -3$  ( attractor )  
 $F$  is strictly positive on the interval:  $0.0119, 0.303, 0.604, 0.573, 0.140, 0.0030$  at  
 $0_{\text{m}} = 0.01, 0.2, 0.5, 0.685, 0.95, 0.999$  .  $0_{\text{m}}$  is monotone increasing with a single stable  
end point.  $0_{\text{m}}L \rightarrow 1$  for every  $0_{\text{m}}L(0)$  in  $(0,1)$  of this system.

### C3 the post-Newtonian limit

4D

a conformally coupled scalar of range  $\lambda = \hbar c/(mc)$  and coupling  $b$  gives  
 $\gamma - 1 = -2 b^2 e^{-r/\lambda} / (1 + 2 b^2 e^{-r/\lambda})$   
with the inverse-square-law floor  $\lambda < 38.6 \mu\text{m}$  ( Lee et al. 2020 ):  
 $r/\lambda = 1.803e13$  at the solar radius ,  $3.876e15$  at 1 AU  
so  $\gamma = 1$  and  $\beta = 1$  to any attainable precision, and the perihelion advance  
42.98 as/cy, the Shapiro delay, the deflection and the redshift follow as outputs.  
This holds whenever the modulus mass clears the floor of D3 below.

### C4 the horizon numbers

4D

$\lambda = 1.09e-52 \text{ m}^{-2}$   $R_{\text{h}} = \sqrt{3/\lambda} = 1.6590e26 \text{ m} = c / (H_0 \sqrt{0_{\text{m}}L})$   
 $T_{\text{dS}} = \hbar c / (2 \pi k_{\text{B}} R_{\text{h}}) = 2.1968e-30 \text{ K}$   
 $\rho_{\text{Lam}} = 3 c^4 / (8 \pi G R_{\text{h}}^2) = 5.2488e-10 \text{ J/m}^3$   
 $E_{\text{Lam}} = (\rho_{\text{Lam}} \hbar c)^{1/4} = 2.2404 \text{ meV}$  measured 2.24 meV  
 $\lambda \text{ LP}^2 = 2.847e-122 = 3 (\text{LP}/R_{\text{h}})^2$  , an identity relating the two scales.  
 $R_{\text{h}}$  is reconstructed from  $H_0$  and  $0_{\text{m}}L$  , so these four lines are a closed consistency set  
on one measured length, not four independent predictions.

## APPENDIX D - the lift and the ladder

the Kaluza-Klein sector, the tower, the modulus floor, and the dimensional counting

### D1 the Kaluza-Klein sector

5D

ansatz  $ds^2 = g_{mn} dx^m dx^n + (dy + A_m dx^m)^2$ ,  $y \sim y + 2\pi R$   
reduce  $S = (1/16\pi G_5) \int d^4x \sqrt{-g_5} R_5$   
 $= (2\pi R / 16\pi G_5) \int d^4x \sqrt{-g} [R_4 - (1/4) F_{mn} F^{mn}]$   
so  $G_4 = G_5/(2\pi R)$ , and with  $A = \sqrt{16\pi G_4} A^a$  and fibre momentum  $p_y = n/R$ ,  
 $q_n = (n/R) \sqrt{16\pi G}$   $\alpha = e^2/4\pi = 4 (l_P / R)^2$   
 $R = 2 l_P / \sqrt{\alpha} = 23.4125 l_P = 3.7841e-34 \text{ m}$ ; back-substitution returns 137.036  
SCOPE:  $q/m = \sqrt{16\pi} = 7.08982$  for every mode on the fibre, independent of  $R$ , so this  
relation fixes  $R$  from  $\alpha$  and  $G$  rather than predicting either. Charged matter of the  
Standard Model is not fibre momentum on this circle; the relation is carried as the  
parameterisation of the fibre, and  $R$  remains an unexplained length.

### D2 the tower and the grain-scale curvature term

5D

KK spectrum  $m_n = |n| / R$ ;  $m_1 = \sqrt{\alpha} m_P / 2 = 5.2147e17 \text{ GeV}$   
(two routes,  $\hbar c/R$  and  $\sqrt{\alpha} m_P/2$ , agree to  $3.1e-10$ )  
Lovelock: the Gauss-Bonnet invariant first becomes dynamical at  $D = 5$ . At the grain scale  
its fractional contribution goes as  $(a/r)^2$ :  
 $1.70e-62$  at  $52 \text{ um}$   $4.59e-71$  at  $1 \text{ m}$   $9.47e-89$  at the solar radius  
the gauge sector of this reduction is electromagnetism itself and is not suppressed;  
only the higher-curvature term is.

### D3 the modulus and its mass floor

5D

the modulus is the canonically normalised fluctuation of the fibre size, with  
 $m^2 = d^2 V_{\text{eff}} / d\phi^2$  at the minimum  
a massless modulus is excluded by the Cassini bound  $\gamma - 1 = (2.1 \pm 2.3)e-5$  by  $\sim 1e4$ ,  
so  $V_{\text{eff}}$  is non-flat. Its range must clear the inverse-square-law bound:  
 $\lambda = \hbar/(mc) < 38.6 \text{ um} \Rightarrow m > \hbar c / 38.6 \text{ um} = 5.112 \text{ meV}$   
this is the floor on which the post-Newtonian result of C3 rests.

### D4 the counting ladder

6D

a massless spin-2 field in  $D$  dimensions carries  $N(D) = D(D-3)/2$  states. Under  $D = 4 + n$ :  
 $2 + 2n + n(n+1)/2 = (4 + 4n + n^2 + n)/2 = (n+1)(n+4)/2 = D(D-3)/2$  QED  
 $D = 4..11$ : 2, 5, 9, 14, 20, 27, 35, 44 - exact at every rung.  
 $D = 5$ :  $5 = 2 + 2 + 1$   $D = 6$ :  $9 = 2 + 4 + 3$   $D = 7$ :  $14 = 2 + 6 + 6$   
a 2-form in  $D$  carries  $C(D-2,2)$  states; at  $D = 6$  that is 6, halved to 3 by self-duality,  
which is the reading of the third entry at that rung.

### D5 the isometry floor and the chirality condition

7D

$G = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ ,  $\dim G = 12$ ;  $H = \text{SU}(2) \times \text{U}(1) \times \text{U}(1)$ ,  $\dim H = 5$   
 $\dim(G/H) = 7 \Rightarrow D = 4 + 7 = 11$ . Minimality is Witten's 1981 theorem, imported.  
chirality condition, at defensible scope: a metric-only reduction on a smooth compact  
manifold without boundary, flux or orbifold fixed points yields no net 4D chirality.  
Orbifolds and fluxes evade it; declining them is a modelling choice of this framework,  
not a theorem, and it is the choice that terminates the ladder.

## APPENDIX E - scope

what the framework establishes, what it establishes conditionally, and what it does not

### E1 established

4D

the coherence gate as the reciprocal identity ( B1 )  
the correction tensor  $\Delta_{mn}$  in closed form, with a symbolic check ( B2 )  
one null cone for photons and tensor modes, for any  $C(x)$  and  $X(x)$  ( B3 )  
scalar stability: no ghost and no gradient instability on the whole sheet ( B4 )  
the sign of the foam force, from the matter coupling: attraction ( B5 )  
 $w = -1/3 - (2/3) \sqrt{0m_L}$  as the exact consequence of horizon saturation ( C1 )  
the late-time attractor, with eigenvalues +1 and -3 ( C2 )  
the horizon consistency set:  $R_h, T_{dS}, \rho_{Lam}, E_{Lam}$  ( C4 )  
the counting ladder identity, exact for  $D = 4$  to 11 ( D4 )

### E2 conditional

5D

$\gamma = \beta = 1$ , and with them Mercury, Shapiro, deflection and redshift ( C3 )  
conditional on the modulus mass clearing 5.112 meV, which requires  $V(X,C)$   
the fibre radius  $R = 2 \text{ lP} / \sqrt{\alpha}$  ( D1 )  
a parameterisation of the fibre in terms of  $\alpha$  and  $G$ , not a prediction of either  
the scalar cone running to  $1/\sqrt{3}$  ( B4 )  
a property of the polynomial and of the lattice, quoted outside the effective range  
the field content read off each rung of the ladder ( D4 )  
the counting is exact; the assignment of fields to it presumes a specific reduction

### E3 OPEN

- 1  $\eta_S$  from grain combinatorics without  $G$ . The pair  $\eta_S = c^3/4G \hbar$  and  $G = c^3/4 \hbar \eta_S$  is exact and locked; an independent count of the grain degrees of freedom is what would turn it into a derivation of the gravitational coupling.
- 2  $V(X,C)$ . Every conditional entry in E2 rests on it, as does the existence of the background  $X = 1 + \chi$  and the late-time recovery of General Relativity.
- 3 the origin of 0.419. The grain length sets  $a, \mu^4$ , the Gauss-Bonnet suppression and the per-grain entropy  $s_0 = a^2/4 \text{ lP}^2 = 0.043890$  nats, so all four inherit it.
- 4 the origin of  $R = 23.4125 \text{ lP}$ , for the reason given in D1.
- 5 horizon saturation is postulated rather than obtained from the action of A2.
- 6 the three-domain weights  $w = 1/2 w_0$  are a modelling choice; no dynamics forces the  $1/2$ .
- 7 an observable below the tower. The scalar dispersion sits at  $5e17 \text{ GeV}$  and the grain-scale curvature term at  $1e-62$ ;  $w(z)$  against supernovae and BAO is the only presently accessible channel, and it is the one that should be run next.
- 8 a single compactification. The fibre of D1, a stabilising geometry for D3 and the 7-coset of D5 are three different internal spaces and are not yet one.

# APPENDIX F - what the mathematics establishes

seventeen independent verification passes; what they settle, and the one calculation that decides the rest

## F1 the verified results, and what class each belongs to

4D

Delta\_mn conformal identity - true of ANY conformal factor, curved background,  $w(t,x)$   
one null cone for photons - conformal invariance; true whether or not foam exists  
 $\lambda_{\text{dabar}} C_{R_S} = 2 \text{ lP}^2$  - holds for every mass over 60 decades: an identity  
42.98 as/cy - General Relativity, confirmed by geodesic integration  
 $N(D) = D(D-3)/2$  -  $SO(D-2)$  representation theory  
 $\eta_S \leftrightarrow G$  - exact rearrangement of a definition

every item above is a mathematical identity, or GR recovered, or a consequence of an assumed postulate. None of them distinguishes a universe with foam from one without. That is not a defect in the algebra. It is what happens when a framework is built to recover known physics: recovering it cannot then be evidence for it.

**STATUS: the chart is CONSISTENT. Consistency is not existence.**

## F2 the one place a number comes out that was not put in

4D

G, alpha,  $H_0$  and  $Om_L$  are inputs.  $R = 23.4125 \text{ lP}$  is G and alpha wearing a length;  
 $E_{\text{Lam}} = 2.24 \text{ meV}$  is  $R_h$  wearing an energy. Those are re-parameterisations.

the exception: given  $Om_L = 0.685$ , saturation returns  $w_0 = -0.885098$  with NO free parameter. LCDM requires -1. That is an output.

measured against DESI DR2 + CMB + Pantheon+ ( $w_0 = -0.838 \pm 0.055$ ): 0.9 sigma.  
the evolution term disagrees: model  $w_a = +0.231$  against  $w_a = -0.62 (+0.22/-0.19)$ , 3.9 sigma.

**STATUS: half passing - which is a real status, and a rare one**

## F3 the path to macro physics, followed through

4D

the chart draws the spine as foam  $\rightarrow \eta_S \rightarrow G \rightarrow$  Einstein. That leg is circular.  
the leg that actually reaches observation is saturation  $\rightarrow w(0m_L) \rightarrow$  growth, and it does not pass through the grain at all. Integrated in full:

|                           |                                                                   |                                        |                              |
|---------------------------|-------------------------------------------------------------------|----------------------------------------|------------------------------|
| age of the universe       | 13.323 Gyr                                                        | against LCDM 13.797                    | ( -0.474 Gyr, 3.4% younger ) |
| $H(z)$ excess             | +5.6% at $z = 1$                                                  | ( BAO measures $H(z)$ to sub-percent ) |                              |
| distance modulus          | -0.012 mag at $z=0.1$ rising to -0.089 at $z=2$                   |                                        |                              |
|                           | the 0.077 mag DIFFERENTIAL is not degenerate with SN calibration  |                                        |                              |
| linear growth             | sigma8 lower by 14.6% at fixed early normalisation                |                                        |                              |
| $f \text{ sigma8 } (z=0)$ | suppressed by 21% ( RSD precision is 5-8% : a 3-4 sigma channel ) |                                        |                              |

these are macro physics, they follow from one postulate rather than a fit, and they are testable with data that already exists.

**STATUS: the path is real; it is not the path the chart draws**

## F4 the grain: the calculation run, and its negative result

3D

5835 closed-form candidates were scanned against  $a = 0.419 \text{ lP}$ . Two land within 0.0005 ( $\pi^{-1} \sqrt{3}^{0.5} = 0.418919$ , 0.019% off ). The local candidate density predicts 1.9 accidental matches at that tolerance on its own, so the hit carries no information.

NEGATIVE RESULT, final at this precision: the origin of 0.419 cannot be found by numerology at three significant figures. It comes from the lattice count or not at all.

**STATUS: closed - a search was run and returned nothing**

## F5 the single number that decides the framework

3D

a plaquette of side  $a$  tiling a horizon reproduces  $S = A/4\text{lP}^2$  only if each plaquette carries exactly

$$s_0 = a^2 / (4 \text{ lP}^2) = 0.0438902500 \text{ nats} \quad \exp(s_0) = 1.0448676744 \text{ states}$$

so the grain is almost entirely frozen: one state plus a 4.49% excess. For comparison a 2-state grain would require  $a = 1.6651 \text{ lP}$ , not 0.419.

a microstate count over grains that returns 0.0438903 nats per plaquette WITHOUT using G would make  $\eta_S$  an output and change the status of every other line on both pages.

Nothing else on the chart tests the foam. This is the whole experiment.

**STATUS: OPEN, and now stated as one number rather than a programme**